REPRESENTATIONS OF REAL REDUCTIVE GROUPS, DIRAC OPERATORS AND RELATED TOPICS

Institut Elie Cartan de Lorraine, site de Metz

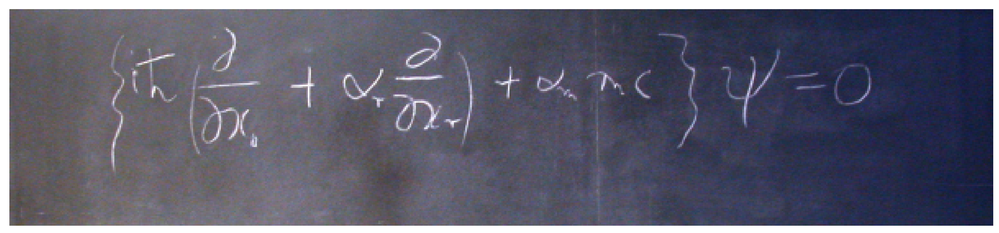
Université de Lorraine – Metz

Amphi Hermite, UFR MIM

10 - 11 octobre 2014

Speakers

Jean-Louis Clerc *(Université de Lorraine - Nancy)*
Lucas Fresse *(Université de Lorraine - Nancy)*
Hrvoje Kraljević *(University of Zagreb)*
Gang Liu *(Université de Lorraine - Metz)*
Pavle Pandžić *(University of Zagreb)*
Ana Prlić *(University of Zagreb)*
Nicolas Prudhon *(Université de Lorraine - Metz)*



Dirac Equation preserved on a chalkboard, Florida State University

http ://fsuspecialcollections.wordpress.com/2014/08/08/happy-birthday-paul-dirac/

Schedule

Friday, October 10th
10h - 11h : *K-structure of $U(\mathfrak{g})$ for $\mathfrak{g} = su(n,1)$ and $\mathfrak{g} = so(n,1)$,* by Hrvoje Kraljević.
11h - 12h : *Dirac induction for non-holomorphic discrete series representations of the group $SU(n,1)$,* by Ana Prlić.
12h -14h : Lunch break.
14h - 15h : *On varieties of partial flags associated to nilpotent elements,* by Lucas Fresse.
15h - 16h : *Branching problem to non-reductive subgroups of a reductive Lie group,* by Gang Liu.
16h15 - 17h15 : *Conformal covariance of the powers of the Dirac operator,* by Jean-Louis Clerc.
19h : Dinner at *Au Détour, 8 rue des Huiliers, 57000 Metz*
Saturday, October 11th
10h - 11h : *Dolbeault cohomology representations and Zuckerman translation functor,* by Nicolas Prudhon.
11h - 12h : *Classifying unitary modules by their Dirac cohomology,* by Pavle Pandžić.

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K-structure of $U(\mathfrak{g})$ for $\mathfrak{g} = su(n,1)$ and $\mathfrak{g} = so(n,1)$, by Hrvoje Kraljević.

Abstract : Let G be the adjoint group of the Lie algebra $\mathfrak{g} = su(n,1)$ or $\mathfrak{g} = so(n,1)$ and let K be its maximal compact subgroup. It will be shown that there exists a K-submodule H of the universal enveloping algebra $U(\mathfrak{g})$ (with respect to the adjoint action of K) such that $U(\mathfrak{g})$ is as a K-module isomorphic to $U(\mathfrak{g})^K \otimes H$ and that the multiplicity of every K-type δ in H is less than or equal to its degree $d(\delta)$. Our conjecture is that it is always equal and for small values of n we will give some evidence in favor of this conjecture. A corollary of our result is that for every finite dimensional K-module V the space of K-invariants $(U(\mathfrak{g}) \otimes V)^K$ is as a $U(\mathfrak{g})^K$-module free of finite rank (less than or) equal to $\dim V$.

Dirac induction for non-holomorphic discrete series representations of the group $SU(n,1)$, by Ana Prlić.

Abstract : In a joint paper, P. Pandžić and D. Renard introduced new notions of Dirac cohomology and homology of a Harish-Chandra module X. If X is unitary or finite-dimensional then they both coincide with the version of Dirac cohomology defined by D. Vogan. The functor of Dirac cohomology has a left adjoint functor and the functor of Dirac homology has a right adjoint functor, which are both called Dirac induction functors. Holomorphic and anti-holomorphic discrete series can be obtained using Dirac induction. Except for those two types of discrete series, the group $SU(n,1)$ has also a third type, neither holomorphic nor anti-holomorphic. We are going to show that representations of the non-holomorphic discrete series of the group $SU(2,1)$ can be constructed via Dirac induction and generalize some results for the group $SU(n,1)$.

On varieties of partial flags associated to nilpotent elements, by Lucas Fresse.

Abstract : In geometric representation theory, an important fact is the relation between the flag variety of a reductive group (i.e., the quotient G/B by a Borel subgroup) and the nilpotent cone of the Lie algebra $Lie(G)$. The most classical result based on this relation is the construction of Springer representations. In this talk, we will rather emphasize the relation between partial flag varieties (i.e. quotients G/P by parabolic subgroups) and the nilpotent cone. Specifically, we will study a family of subvarieties associated to nilpotent elements. We will emphasize their role in representation theory and certain of their geometric properties.

Branching problem to non-reductive subgroups of a reductive Lie group, by Gang Liu.

Abstract : Let $H \subset G$ be Lie groups with G reductive. Let π be an irreducible unitary representation of G. We want to study la restriction of π to H, $\pi|_H$. If H is compact, fundamental facts can go back to Weyl and Harish-Chandra, and the last major results were due to Schmid and Hecht on "Blattner's conjecture" in the 1970s. Since that time, there has been extensive work on the case where H and G are both reductive. In this context, major progress has been made especially by Kobayashi and his collaborators. However, when H is non-reductive, many things are unknown. This is especially the case when H is a parabolic subgroup of G and π is "non-holomorphic" (e.g. "non-holomorphic" discrete series). In my talk, I will try to illustrate this problem via $G = SU(2,1)$ and $G = Spin(4,1)$. If time permits, I will discuss a possible strategy for attacking this problem in general. In particular, we believe that Dirac operators could play an important role.

Conformal covariance of the powers of the Dirac operator, by Jean-Louis Clerc.

Abstract : The odd powers of the Dirac operator on the Euclidean space satisfy a covariance property under the action of the conformal group. I will present a new proof (joint work with B. Ørsted), interpreting the odd powers of the Dirac operator as residues of the meromorphic family of intertwining operators for the spinorial principal series.

Dolbeault cohomology representations and Zuckerman translation functor, by Nicolas Prudhon.

Abstract : It is known that Zuckerman translation functor commutes with cohomological induction, the algebraic counterpart of Dolbeault cohomology representations. We show how to recover this result in the geometric context, using techniques adapted from those developed by Huang and Pandžić in their work on Dirac cohomology.

Classifying unitary modules by their Dirac cohomology, by Pavle Pandžić.

Abstract : In this ongoing work with Jing-Song Huang (with help from David Vogan), we address the question of determining $A_{\mathfrak{q}}(\lambda)$-modules by their Dirac cohomology. The answer is that almost always such a module is uniquely determined by its Dirac cohomology, but there are certain series of exceptions. We can list the exceptions in an organized way.